FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed**.

Q. No. 1. **(a)** If $\psi = Sin\frac{(kr)}{r}$, then show that $\nabla^2\psi + k^2\psi = 0$. **(10)**

(b) Calculate the Line Integral $\int_c A.dr,\ where\ A = \frac{-yi + xj}{x^2 + y^2}$, and the curve C is given by the equations $x^2 + y^2 = a^2\ \ and\ \ Z = 0$. **(10)**

Q. No. 2. **(a)** Forces of magnitude P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of sides **a**, and forces each of magnitude $(8\sqrt{2})$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and distance of its line of action from A. **(10)**

(b) A uniform ladder, of length 70 feet, rests against a vertical wall with which it makes an angle of 45^0, the coefficient of friction between the ladder and the wall and the ground respectively being $\frac{1}{3}$ and $\frac{1}{2}$. If a man, whose weight is one half that of the ladder, ascends the ladder, where will he be when the ladder slips? **(10)**

Q. No. 3. **(a)** A particle moves in a straight line with an acceleration kv^3. If its initial velocity is u, find the velocity and the time spent when the particle has travelled a distance x. **(10)**

(b) Derive the Tangential and Normal components of the velocity and acceleration. **(10)**

Q. No. 4. **(a)** Solve the following Cauchy- Euler Equation **(10)**

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0.$$

(b) Convert the following Bernoulli Differential Equation into standard form and then solve. **(10)**

$$\frac{dy}{dx} + \left(\frac{xy}{1-x^2}\right) = xy^{\frac{1}{2}}.$$

Q. No. 5. **(a)** Convert the following Ordinary Differential Equation into standard form and then solve using Method of Variation of Parameters. **(10)**

$$x^2y'' - 3xy' + 3y = 2x^4e^x$$

(b) Check whether the following Ordinary Differential Equation is an Exact Equation or not. If yes, then solve. **(10)**

$$(3x^2y + 2)dx + (x^3 + y)dy = 0$$

Q. No. 6. **(a)** Find the Fourier Series of f on the given interval. **(10)**

$$f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0\le x<\pi\end{cases}$$

(b) Solve the following Partial Differential Equation subject to the conditions given. **(10)**

$$a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}, \qquad 0<x<L, \qquad t>0,$$

$$u(0,t)=0, \qquad u(L,t)=0, \qquad t>0,$$

$$u(x,0)=f(x),\ \frac{\partial u}{\partial t}=g(x)[at\ time\ t=0], \qquad and \quad 0<x<L.$$

Q. No. 7. **(a)** Use Newton-Raphson method to find solution accurate to within 10^{-4} for the non-linear equation. **(10)**

$$x^3-2x^2-5=0, \qquad I=[1,4]$$

(b) Use Lagrange Interpolating polynomial of degree two to approximate $f(8.4)$, If **(10)**

$$f(8.1)=16.94410,\quad f(8.3)=17.56492,\quad f(8.6)=18.50515,\quad f(8.7)=18.82091.$$

Q. No. 8. **(a)** Approximate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule and Simpson's rule with $n=4$. **(10)**

Also compare your results with the exact value of the integral.

(b) Use Euler's method to approximate the solution of the following initial value problem. **(10)**

$$y'=\frac{1+y}{t}, \qquad 1\le t\le 2, \qquad with\ \ y(1)=2, \qquad h=0.25$$

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